Let R grow, and ν_1, ν_2 be fixed. the domain $]0; \alpha_1[$ of values α, for which the initial deviation of the slip lines from the interfacial boundary of the media holds, diminishes. For fixed α the angle of the initial deviation of the slip lines from the interfacial boundary of the media diminishes. That value of α diminishes, starting with which the slip lines do not deviate from the interfacial boundary of the media.

Values of the slope of the slip lines to the interfacial boundary of the media are presented in degrees in the lower part of the table for certain values of α and R ($\nu_1 = 0.333$, $\nu_2 = 0.250$).

REFERENCES

1. CHEREPANOV G.P., Mechanics of Brittle Fracture, Nauka, Moscow, 1974.
2. UFLYAND YA.S., Integral Transforms in Problems of Elasticity Theory, Nauka, Leningrad, 1967.
3. GAKHOV F.D., Boundary Value Problems, Nauka, Moscow, 1977.
4. NOBLE B., Application of the Wiener-Hopf Method for Solving Partial Differential Equations, IIL, Moscow, 1962.
5. CHEREPANOV G.P., On the ambiguity problem in plasticity theory, Dokl. Akad. Nauk SSSR, 218, 4, 1974.

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STABILITY OF MOTION OF LINEAR SYSTEMS RELATIVE TO SOME OF THE VARIABLES*

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Using the method of Lyapunov functions, we obtain the sufficient conditions for asymptotic stability of linear systems with constant coefficients, with respect to some of the variables.

Suppose we have a system of linear differential equations of perturbed motion (A is a constant $(n \times n)$ matrix):

$$\mathbf{x}^{\cdot} = A\mathbf{x};\quad \mathbf{x} = (y_1, \ldots, y_m, z_1, \ldots, z_p) = (\mathbf{y}, \mathbf{z})$$
$$m > 0,\quad p \geqslant 0,\quad n = m + p \tag{1}$$

We consider the problem of the asymptotic y-stability of the unperturbed motion $\mathbf{x} = 0$ /1-4/.

Let B and B_ε be symmetric $(n \times n)$ matrices, and let $B^{(i)}$ $(i = 1, \ldots, 4)$ be matrix blocks of orders $m \times m$, $m \times p$, $p \times m$, $p \times p$, respectively, such that (E_m denotes the $(m \times m)$ identity matrix)

$$B = \left\| \begin{matrix} B^{(1)} & B^{(2)} \\ B^{(3)} & B^{(4)} \end{matrix} \right\|,\quad B_\varepsilon = \left\| \begin{matrix} B^{(1)} - \varepsilon E_m & B^{(2)} \\ B^{(3)} & B^{(4)} \end{matrix} \right\|$$
$$\varepsilon = \text{const} > 0$$

The quadratic form $v = v(\mathbf{x})$, $v(0) = 0$, is said to be: 1) positive semidefinite in all variables, if $v(\mathbf{x}) \geqslant 0$ for all $\|\mathbf{x}\| < \infty$ /5/; 2) y-positive semidefinite if $v(\mathbf{x}) \geqslant a(\|\mathbf{y}\|)$ for all $\|\mathbf{x}\| < \infty$ /2/, where $a(r)$ is a continuous and monotone increasing function of $r \in [0, \infty)$, $a(0) = 0$.

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Verification that $v(\mathbf{x})$ is y-positive semidefinite can be reduced to verification of Sylvester's conditions $\Delta_{ii} \geqslant 0$ $(i = 1, \ldots, n)$ for the auxiliary form $v^*(\mathbf{x}) = \mathbf{x}^T B_\varepsilon \mathbf{x}$ to be positive semidefinite in all its variables (i.e., for the matrix B_ε to be positive semidefinite). Here Δ_{ii} are the principal diagonal minors of the matrix B_ε.

Lemma. The form $v(\mathbf{x}) = \mathbf{x}^T B\mathbf{x}$ is y-positive semidefinite if and only if the form $v^*(\mathbf{x})$ (the matrix B_ε) is positive semidefinite for sufficiently small ε.

Together with B, B_ε, we shall consider matrices C, C_ε of the same order and structure.

Theorem 1. If the Lyapunov matrix equation $A^T B + BA = -C$ is solvable in the class of symmetric constant matrices B, C such that B_ε (C_ε) is positive (negative) semidefinite for sufficiently small ε, then the motion $\mathbf{x} = 0$ of system (1) is asymptotically y-stable.

Proof. By the lemma, there is a quadratic y-positive semidefinite form $v = \mathbf{x}^T B\mathbf{x}$ whose derivative $v^\cdot$ along trajectories of system (1) is y-negative semidefinite. But since $v^\cdot \leqslant 0$, the right-hand sides of the first m equations of system (1) are bounded, and consequently /2/ the motion $\mathbf{x} = 0$ is asymptotically y-stable.

Theorem 2. If constants $c_i > 0$ $(i = 1, \ldots, m)$, $c_j \geqslant 0$ $(j = m + 1, \ldots, n)$ exist, satisfying the conditions

$$l_i = c_i a_{ii} + \sum_{k=1,\ k\neq i}^{n} c_k |a_{ki}| < 0 \quad (i = 1, \ldots, m), \tag{2}$$

$$l_j \leqslant 0 \quad (j = m + 1, \ldots, n), \quad A = (a_{kr}) \ (k, r = 1, \ldots, n)$$

then the motion $\mathbf{x} = 0$ of system (1) is asymptotically y-stable.

The proof, using the function $V = c_1|x_1| + \ldots + c_n|x_n|$, follows the same lines as that of Theorem 1, relying on conditions (2). Indeed, we have

$$V \geqslant \sum_{i=1}^{m} c_i |x_i| \triangleq a(\|\mathbf{y}\|), \quad D^+V = \sum_{k=1}^{n} c_k \operatorname{sign} x_k \Big[\sum_{r=1}^{n} a_{kr} x_r\Big] \leqslant \sum_{i=1}^{m} l_i |x_i| \triangleq -b(\|\mathbf{y}\|)$$

(D^+ is the upper Dini derivative /2/, and $b(r)$ a function of the same class as $a(r)$). Consequently /2/, the motion $\mathbf{x} = 0$ of system (1) is asymptotically y-stable.

REFERENCES

1. RUMYANTSEV V.V., On the stability of motion with respect to some of the variables. Vestnik Moskov. Gos. Univ., Ser. Mat., Mekh., Khim., Fiz., Astron., 4, 1957.
2. OZIRANER A.S. and RUMYANTSEV V.V., The method of Lyapunov functions in the problem of the stability of motion with respect to some of the variables. Prikl. Mat. Mekh., 36, 2, 1972.
3. VOROTNIKOV V.I. and PROKOP'YEV V.P.,On the stability of motion with respect to some of the variables for linear systems. Prikl. Mat. Mekh., 42, 2, 1978.
4. KRIVOSHEYEV YU.A. and LUTSENKO A.V., On the stability of motion with respect to some of the variables for linear systems with constant and almost constant matrices. Prikl. Mat. Mekh., 44, 2, 1980.
5. KRASOVSKII N.N., Some Problems in the Theory of Stability of Motion, Fizmatgiz, Moscow, 1959.

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